

# THE 10th CUMBERLAND CONFERENCE

MAY 16 - 18, 1997

EMORY UNIVERSITY  
ATLANTA, GA

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The Office of Naval Research,  
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The Cumberland Conference is designed to bring together discrete mathematicians and computer scientists to discuss problems and results of mutual interest.

The conference has established a strong reputation for providing quality speakers and for vigorously promoting the applications of discrete mathematics, especially in the area of computer science.

The goal is to provide an environment with ample time for participants to hear and discuss problems of mutual interest.

Conference Organizer: Ron Gould

Dept. of Math & CS  
Emory University  
Atlanta, GA 30322 or rg@mathcs.emory.edu

Conference material such as schedule, hotel information and other local info, has been available on our web page at:

[www.dc.peachnet.edu/~unixcorn/cmbrconf.htm](http://www.dc.peachnet.edu/~unixcorn/cmbrconf.htm)

We thank DeKalb College and Madelyn Gould for maintaining this page.

Included in this booklet are:

Conference Schedule

Abstracts of Talks

Local Restaurant Guide

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**Cumberland Schedule - Friday, May 16**  
**128 Anatomy and Physiology Building**

Time	Speaker	Title
12:00- 1:00	REGISTRATION	
1:00- 2:00	Brian Alspach	<i>Decomposing Cayley Graphs</i>
2:05- 2:35	Bruce Landman	<i>On Restricting the Set of Common Differences in van der Waerden's Theorem</i>
2:40- 3:10	Andre Kézdy	<i>Consensus and indecomposable hypergraphs</i>
3:10- 3:40	BREAK	
3:40- 4:10	Charles Cooke	<i>The Hadamard Matroid and Generalized Hadamard Codes</i>
4:15- 4:45	Doug West	<i>Some Results on Generalized Coloring</i>
4:50- 5:20	T. J. Reid	<i>A Non-Planar Version of Tutte's Wheels Theorem</i>
5:25- 5:55	Peter Slater	<i>Acquisition and Consolidation Parameters for Graphs</i>
6:00- 8:00	RECEPTION	Cox Hall - 3rd Floor Rooms 1&2

**Cumberland Schedule - Saturday Morning - May 17**  
**128 Anatomy and Physiology Building**

Time	Speaker	Title
8:00 - 8:30	Coffee	
8:30 - 9:00	Cheng Zhao	<i>Extremal <math>n</math>-Colorable Subgraphs with Index Domain Alignment Restrictions</i>
9:05 - 9:35	Heiko Harborth	<i>Weakened Multicolor Ramsey Numbers</i>
9:40 -10:10	Richard Schelp	<i>An Extremal Problem for Vertex Colorings with a Distance Restriction</i>
10:10-10:40	BREAK	
10:40-11:10	Carla Savage	<i>Balanced Gray Codes</i>
11:15-12:15	Peter Winkler	<i>Constraint Graphs and Phase Transitions: A Point Process and a Pursuit Game</i>
12:15-2:00	LUNCH	

**Cumberland Conference Schedule - Saturday Afternoon - May 17**  
**128 Anatomy and Physiology Building**

Time	Speaker	Title
2:00-2:30	John Bruno	<i>Scheduling Two-Point Stochastic Jobs to Minimize the Makespan</i>
2:35-3:05	Michael Plummer	<i>On The Connectivity of Graphs Embedded in Surfaces</i>
3:10-3:40	Xiaoya Zha	<i>Connectivity Interpolation For Genus Embedded Graphs</i>
3:40- 4:00	BREAK	
4:00-5:00	Peter Winkler	<i>Part II: Fertile Graphs and Branching Random Walks</i>
5:05-5:35	Robert Robinson	<i>Hamilton Cycles Containing Randomly Selected Edges in Random Cubic Graphs</i>
5:40- 6:10	Brendan McKay	<i>Constructive Enumeration of Cubic Graphs</i>

**Cumberland Schedule - Sunday May 18**  
**128 Anatomy and Physiology Building**

Time	Speaker	Title
8:00 - 8:30	Coffee	
8:30 - 9:00	Jeno Lehel	<i>Tough Enough Chordal Graphs are Hamiltonian</i>
9:05 - 9:35	Prasad Tetali	<i>Random Sampling of Combinatorial Structures</i>
9:40 -10:10	Guantao Chen	<i>Intersections of Longest Cycles in <math>k</math>-Connected Graphs</i>
10:15-10:45	Charles Suffel	<i>Component Size Connectivity</i>
10:45-11:05	BREAK	
11:05-11:35	Terry McKee	<i>Neighborhood Trees and Chordal Bipartite Graphs</i>
11:40-12:40	Howard Karloff	<i>How Big a Planar Graph Can You Find in that Graph?</i>

**ABSTRACTS FOR THE  
10TH CUMBERLAND CONFERENCE  
EMORY UNIVERSITY  
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**Friday, May 16**

**Brian Alspach, Simon Fraser University**  
*Decomposing Cayley Graphs*

Abstract: I shall present a survey on Cayley graph decompositions. In particular, Hamilton decompositions, 1-factorizations and isomorphic factorizations will be considered.

**Bruce Landman, University of North Carolina at Greensboro**  
(joint work with Tom C. Brown and Ronald L. Graham).

*On Restricting the Set of Common Differences  
in van der Waerden's Theorem.*

Abstract: We want to know for which sets  $A$ , of positive integers, the following statement holds: for all positive integers  $r$  and  $k$ , there exists a positive integer  $n = f(k, r)$  such that for every  $r$ -coloring of  $[1, n]$  there exists a monochromatic  $k$ -term arithmetic progression whose common difference belongs to  $A$ . Any subset of the set of positive integers having the above property is called *large*. A set having this property for a specific fixed  $r$  is called  $r$ -*large*. We give some necessary conditions for a set to be large, including the fact that every large set must contain an infinite number of multiples of each positive integer. Also, no large set  $\{a_n : n = 1, 2, \dots\}$  can have  $\liminf_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} > 1$ . Sufficient conditions for a set to be large are also given. Among these are two results which extend a recent result of Bergelson and Leibman which says that the range of any polynomial with integer coefficients, leading coefficient positive, and zero constant term, is a large set. We also show that any set containing  $n$ -cubes for arbitrarily large  $n$ , is a large set. Results involving the connection between the notions of "large" and "2-large" are given. Several open questions and a conjecture are presented.

Friday, May 16

**Andre Kezdy, University of Louisville**

with J. Lehel and R. Powers

*Consensus and Indecomposable Hypergraphs*

Abstract: Motivated by consensus problems, we introduce a hypergraph model to study problems of aggregating data. The model leads naturally to the Helly property and indecomposable hypergraphs. We mention several results, open problems, and conjectures.

**C.H. Cooke, Old Dominion University**

with Iem Heng

*The Hadamard Matroid and Generalized Hadamard Codes*

Abstract: For primes  $p > 2$  the generalized Hadamard matrix  $H(p, pt)$  can be expressed as  $H = x^A$ , where the notation means  $h_{ij} = x^{a_{ij}}$ . It is seen that the Hadamard exponent  $A$  is the matrix representation of the uniform matroid  $U_{q,q}$ , where  $q = pt$ , in a non-standard independence space. An anomaly in the single element extensions of such a matroid is considered. It is also seen that the row vectors of  $A$  represent a q-ary nonlinear error correcting code whose (equi-distant) code words have minimum distance  $d = (p - 1)t$ . The code can be extended so as to possess  $N = p^2t$  code words of length  $pt - 1$ .

**Douglas B. West, University of Illinois**

*Some Results on Generalized Coloring*

Let  $P$  be a family of graphs closed under the taking of subgraphs and disjoint unions. The  $P$ -chromatic number of a graph  $G$ , written  $\chi_P(G)$ , is the minimum  $k$  such that  $V(G)$  can be partitioned into  $k$  sets each inducing a subgraph of  $G$  belonging to  $P$ . When  $P = \{\bar{K}_n\}$ , the  $P$ -chromatic number is the ordinary chromatic number, and in general  $\chi_P(G) \leq \chi(G)$ . This talk will survey results on  $P$ -chromatic numbers by Peter Mihók, Chris Hartman, and others, including generalizations of Brooks' Theorem and bounds on  $P$ -chromatic numbers for planar graphs and other classes. We consider, for example, the family  $P$  consisting of graphs whose components are paths.

**Friday, May 16**

**Talmage James Reid, University of Mississippi**  
with Haidong Wu-Southern University

*A non-planar version of Tutte's Wheels Theorem*

**Abstract:** Tutte's Wheels Theorem states that a 3-connected graph  $G$  with at least 4 vertices that is not a wheel contains either an edge  $e$  such that the deletion of  $e$  from  $G$  leaves a 3-connected graph or an edge  $f$  such that the contraction of  $f$  from  $G$  leaves a graph which is 3-connected and simple. The edges  $e$  and  $f$  above are said to be *deletable* and *simple-contractible*, respectively. We extend the Wheels Theorem by showing that if  $G$  is non-planar, then it contains either at least 1 deletable edge or at least 6 simple-contractible edges. Several other results on deletable and simple-contractible edges in 3-connected graphs are given.

**Peter J. Slater - UAH**

*Acquisition and consolidation parameters for graphs*

**Abstract:** The operation "consolidation" is defined on a graph  $G$  for which each vertex has a nonnegative weight assigned to it. In a consolidation a vertex transfers some or all of its weight to an adjacent vertex with equal or greater value. If all consolidations in a sequence of consolidations reduce the value of a participating vertex to zero, and the starting value of each vertex is one, this is called an acquisition sequence. Of interest here is the minimum possible number  $c(G)$  (respectively,  $a(G)$ ) of resulting vertices with nonzero values in a consolidation (respectively, acquisition) sequence. Theoretical and computational results are presented.

**Saturday, May 17**

**Cheng Zhao, Indiana State University**

*Extremal  $n$ -colorable subgraphs with index domain alignment restrictions*

**Abstract:** Programming distributed-memory machine requires partitioning of the program data structures and distributing of them to the local storage of each individual processor. This problem has been formulated as the index domain alignment problem, that is, to partition the vertex set of a weighted graph under some restrictions. In this paper, we will focus on this problem and introduce other related graph partitioning problems. We formulate the mathematical programming models for these graph partitioning problems and show that all these problems are NP-complete. This talk will address structural as well algorithmic issues.

Saturday, May 17

**Heiko Harborth, Techn. Univ. Braunschweig, Germany**

*Weakened multicolor Ramsey numbers*

Abstract: Ramsey numbers  $R = R(G)$  determine the smallest complete graph  $K(R)$  such that every  $t$ -coloring of the edges of  $K(R)$  contains a monochromatic copy of a given graph  $G$ . - Here first results are discussed for the weaker numbers  $R(s, t; G)$  where every  $t$ -coloring has to contain a copy of  $G$  with at most  $s$  colors only.

**R.H. Schelp, University of Memphis**

*An Extremal Problem for Vertex Colorings with a Distance Restriction*

Abstract: Let  $d, k$  be any two positive integers with  $k > d > 0$ . A  $k$ -coloring of a graph  $G$  is considered such that the distance between each pair of vertices in the same color class is at least  $d$ . Such graphs are said to be  $(k, d)$ -colorable. The object of this paper is to determine the maximum size of  $(k, 3)$ -colorable,  $(k, 4)$ -colorable, and  $(k, k-1)$ -colorable graphs. Sharp results are obtained for both  $(k, 3)$ -colorable and  $(k, k-1)$ -colorable graphs while the results obtained for  $(k, 4)$ -colorable graphs are close to the truth.

This research was done jointly with Guantao Chen and Andras Gyarfas.

**Carla Savage, North Carolina State University**

*Balanced Gray Codes*

Abstract: An  $n$ -bit Gray code is an exhaustive listing of all  $n$ -bit strings in such a way that successive strings differ only in one bit, or, equivalently, a Hamilton path in the  $n$ -cube. A balanced Gray code is one in which the bit changes are distributed as equally as possible among the  $n$  bit positions. Although Wagner and West proved in 1991 that balanced Gray code schemes exist when  $n$  is a power of 2, the question for general  $n$  remained open since 1980 when it first attracted attention.

We show in this talk that balanced  $n$ -bit Gray codes can be constructed for all positive integers  $n$ . The strategy used is to prove the existence of a certain subsequence which will allow successful use of the construction proposed by Robinson and Cohn in 1981. This is joint work with Girish Bhat.

Saturday, May 17

**Peter Winkler, Bell Laboratories**

*Constraint Graphs and Phase Transitions*

**Abstract:** A ‘hard constraint’ model in statistical mechanics is a system whose global behavior is determined by forbidding certain local configurations. Such a system is described graph-theoretically as a space of random homomorphisms from a large graph  $G$  to a fixed constraint graph  $H$ . It turns out that the qualitative behavior of such a system depends on certain properties of the constraint graph. In these two talks we present a graph-theorist’s introduction to this model, including recent results with Graham Brightwell of the London School of Economics. No knowledge of physics will be assumed, and we will present many more pictures than proofs.

*Part I: A Point Process and a Pursuit Game*

We define the notion of a Gibbs measure for random homomorphisms and describe one famous example, the ‘hard-core gas model’, a.k.a. random independent sets in a graph. We then connect the Gibbs measures with stationary distributions of a Markov process, and with cop-win and robber-win graphs.

*Part II: Fertile Graphs and Branching Random Walks*

Here we concentrate on the case where  $G$  is a regular tree, showing that ‘nice’ Gibbs measures correspond to random walks on the constraint graph. Constraint graphs are then classified according to whether there can be a phase transition involving more than one random walk with the same local properties.

**John Bruno, UCSB**

*Scheduling Two-Point Stochastic Jobs to Minimize the Makespan*

**Abstract:** Simple optimal policies are known for the problem of scheduling jobs to minimize the makespan on parallel machines when the job running-time distribution has a monotone hazard rate. But no such policy appears to be known in general. We investigate the general problem by adopting two-point running-time distributions, the simplest discrete distributions not having monotone hazard rates. We derive a policy that gives an explicit, compact solution to this problem and prove its optimality.

Saturday, May 17

Michael D. Plummer, Vanderbilt University  
with Xiaoya Zha

*On The Connectivity of Graphs Embedded in Surfaces*

Abstract: Define the **maximum connectivity**  $\kappa_{\max}$  of a surface (orientable or non-orientable) to be the maximum connectivity of any graph embeddable in that surface and the **genus connectivity**  $\kappa_{\text{gen}}$  of the surface to be the maximum connectivity of any graph which *genus* embeds in that surface.

Cook has found  $\kappa_{\max}$  for each surface. In this talk we will discuss some problems related to  $\kappa_{\text{gen}}$ , such as the relation between  $\kappa_{\max}$  and  $\kappa_{\text{gen}}$ , determination of  $\kappa_{\text{gen}}$  for each surface, uniqueness of graphs attaining  $\kappa_{\text{gen}}$  and the spectrum of values of genera in the intervals  $[\gamma(K_n) + 1, \gamma(K_{n+1})]$  and  $[\bar{\gamma}(K_n) + 1, \bar{\gamma}(K_{n+1})]$  with respect to their genus and maximum connectivities.

Xiaoya Zha, Vanderbilt University  
with Michael D. Plummer

*Connectivity Interpolation for Genus Embedded Graphs*

Abstract: Define the **maximum connectivity**  $\kappa_{\max}$  of a surface (orientable or non-orientable) to be the maximum connectivity of any graph embeddable in that surface and the **genus connectivity**  $\kappa_{\text{gen}}$  of the surface to be the maximum connectivity of any graph which *genus* embeds in the surface.

Let  $n$  be a positive integer and let  $\Sigma$  be a surface. It is easy to decide if there is an  $n$ -connected graph embeddable in surface  $\Sigma$ . However, it seems difficult to decide if there is an  $n$ -connected graph which *genus* embeds in  $\Sigma$ . In this talk we show that, for  $n \in [1, (\sqrt{2}/2) \kappa_{\max}]$ , the above problem has a positive answer.

Robert Robinson, Univeristy of Georgia  
*Hamilton Cycles Containing Randomly Selected Edges  
in Random Cubic Graphs*

Abstract: This is joint work with N. C. Wormald. Previously we showed that almost all labeled cubic graphs are hamiltonian. This result is now generalized to situations in which a set of edges which the cycle must contain is randomly specified in advance. One corollary is that almost all claw-free cubic graphs are hamiltonian.

Saturday, May 17

**Brendan D McKay, Australian National University**

*Constructive Enumeration of Cubic Graphs*

**Abstract:** We report on recent advances in algorithms for generating non-isomorphic cubic graphs of various kinds. For ordinary or bipartite cubic graphs, Shiher Sanjmyatav and the speaker have a new algorithm that is twice as fast as the previously best method. For planar 3-connected graphs, Gunnar Brinkmann and the speaker have an algorithm which is very much faster. Both algorithms utilise a similar method for isomorph rejection. Other applications of the general method will be mentioned.

Sunday, May 18

**J. Lehel, University of Louisville**

with G. Chen, M. S. Jacobson and A.E. Kézdy

*Tough Enough Chordal Graphs are Hamiltonian*

**Abstract:** A connected graph is  $t$ -tough if one has to remove at least  $tk$  points to disconnect the graph into  $k$  connected components. We proved recently that  $t = 18$  implies the existence of a hamiltonian cycle in chordal graphs. In the talk the basic tools of the proof will be presented: a structured elimination ordering on chordal graphs, the Hall-type neighborhood condition derived from large toughness, and the basic path handling procedures used in the hamiltonian cycle algorithm.

**Prasad Tetali, Georgia Tech**

*Random Sampling of Combinatorial Structures*

**Abstract:** The speaker intends to present a summary of his recent contributions to the topic of "rapidly mixing Markov chains." The summary includes simple Markov chain algorithms for generating, uniformly at random, (i) tournaments with a given score vector, (ii) Euler tours in an undirected Eulerian graph, and (iii) several intriguing open questions.

**Guantao Chen, Georgia State University**

with Ralph J. Faudree and Ronald J. Gould

*Intersections of Longest Cycles in  $k$ -Connected Graphs*

**Abstract:** Let  $G$  be a  $k$ -connected graph, where  $k \geq 2$ . S. Smith conjectured that every two longest cycles of  $G$  have at least  $k$  vertices in common. In this talk, we show some progress on this conjecture.

Sunday, May 18

**Charlie Suffel, Stevens Institute of Technology**

*Component Size Connectivity*

**Abstract:** In applications, such as multiprocessor networks, where nodes are susceptible to failure but edges aren't, the traditional measure of vulnerability, namely connectivity, is inappropriate. In such cases it is more important for the surviving subgraph to contain a "large" connected piece, than for it to be connected. We introduce a new graph invariant, called component size connectivity, which addresses this issue. More precisely, given a positive integer  $k$ , the component size connectivity of a graph  $G$ , denoted by  $\kappa_c^{(k)}(G)$ , is the minimum number of nodes which upon removal reduces the graph to one having no component of order  $k$  or more. We discuss basic properties of this invariant, computational complexity and algorithmic issues, the relationship between  $\kappa$  and  $\kappa_c^{(k)}$  and, in particular, extremal and realizability questions concerning them.

**Terry McKee, Wright State University**

*Neighborhood Trees and Chordal Bipartite Graphs*

**Abstract:** "Chordal bipartite graphs" are usually defined to be bipartite graphs with no induced cycles other than 4-gons. While these graphs have been well-studied and have serious applications, I always found them to be somewhat artificial. But suddenly I'm a fan.

I'll show how simply replacing maximal complete subgraphs with open neighborhoods allows the elegant "clique tree" approach to chordal graph theory to be imitated using "neighborhood trees," resulting in a new vertex-based approach to chordal bipartite graphs. I'll also mention related families of graphs corresponding to "closed neighborhood trees," "neighborhood paths," etc.

**Howard Karloff, Georgia Tech**

*How Big a Planar Graph Can You Find in that Graph?*

**Abstract:** For applications such as graph drawing, circuit layout, and facility layout, it is often useful to find the largest planar subgraph of a given nonplanar graph  $G$ . I will discuss new approximation algorithms for this NP-Hard problem. These algorithms run in polynomial time and produce a planar subgraph of  $G$  whose size is at least a fraction  $c$  of the size of the (unknown) largest planar subgraph of  $G$ ; the larger  $c$  is, the better. Obtaining an algorithm with  $c=1/3$  is trivial (use a spanning tree). We will discuss a nontrivial approximation algorithm that gets a fraction of  $1/3+\epsilon$  for a fixed positive  $\epsilon$ . We will also discuss a new algorithm for finding heavy planar subgraphs in WEIGHTED graphs.

This talk is based on joint work with Calinescu, Fernandes, Finkler, and Zelikovsky.

# REPORT DOCUMENTATION PAGE

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